Cautionary Tales

Part X **RESONANCE OF SPRINGS AND SPRING/MASS SYSTEMS**

As most readers will be aware, resonance is a destructive phenomenon which can occur in springs subject to cyclic loading, and which generates premature failures in many instances. The cyclic action of the force being applied to the spring can produce uncontrollable and highly destructive sympathetic vibrations within the spring. Spring designers need to be aware of this potential problem, and be able to predict the critical speeds at which resonance will occur.

In all computer-aided design programs for compression springs, a formula is incorporated that calculates the natural frequency of the spring itself. This is the frequency at which the mass of the spring and its stiffness causes resonance and surge within the spring coils. Unless the spring is extremely long, with a large number of coils and low stiffness, the natural frequency determined by the formula is usually extremely high. For example, in the following typical case the natural frequency is 33,160 cycles per minute:

Material	-	cold drawn carbon steel to ASTM A228
Section Size	-	2 mm
Outside diameter	-	20 mm
Total coils	-	6
End type	-	closed and ground ends
Free length	-	30 mm
Natural frequency	-	33,160 cycles per minute

The rule of thumb that *IST* would employ to ensure that this spring does not resonate significantly in use (or, in our case, on fatigue test) would be to ensure that the applied frequency is less than one thirteenth of the natural frequency. That is to say, the operating or test speed of this spring should not exceed 2550 cycles per minute, and then it would be safe to assume that the calculated operating stresses are not significantly increased by resonance effects. It may seem that one thirteenth is a strange and arbitrary value to use, but it was derived experimentally, and has been proved over many years to be appropriate.

In situations where the spring is required to carry a mass attached to its moving end, the natural frequency of the combined spring/mass system is dramatically reduced, and is likely to lead to problems in many more practical situations at significantly lower operating speeds. For example, in the case of the above spring, the effect of the addition of a 1kg mass at one end of the spring is to dramatically reduce the natural frequency from 33,160 cycles per minute to 784 cycles per minute. It can be seen that the scope for resonance problems in the spring/mass application is dramatically greater than for the spring alone.

Unfortunately, the formula for determining the natural frequency of a spring with supported mass is not given in spring design standards or most CAD programs, but is contained in good Mechanics of Machines text books. The formula is as follows:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{S}{M + m_{/3}}} \quad (Hz)$$

f_n = natural frequency of combined spring/mass system (Hz)

S = stiffness of spring (N/mm)

M = supported mass (kg)

m = mass of spring (kg)

The units of Hertz are cycles per second, and to convert this to cycles per minute, simply multiply the result by 60.

The warning to spring designers is clear; when considering resonance problems in cyclic loading situations, it is vital to establish whether the spring is required to support or act against a movable mass in service. If this is the case, then the formula given above must be used, and this will predict resonance speeds significantly lower than those given by your CAD program, which is valid only for the spring itself. Failure to recognise a risk of resonance or surging could result in failure, a typical compression spring that failed in this way is shown in figure 1.

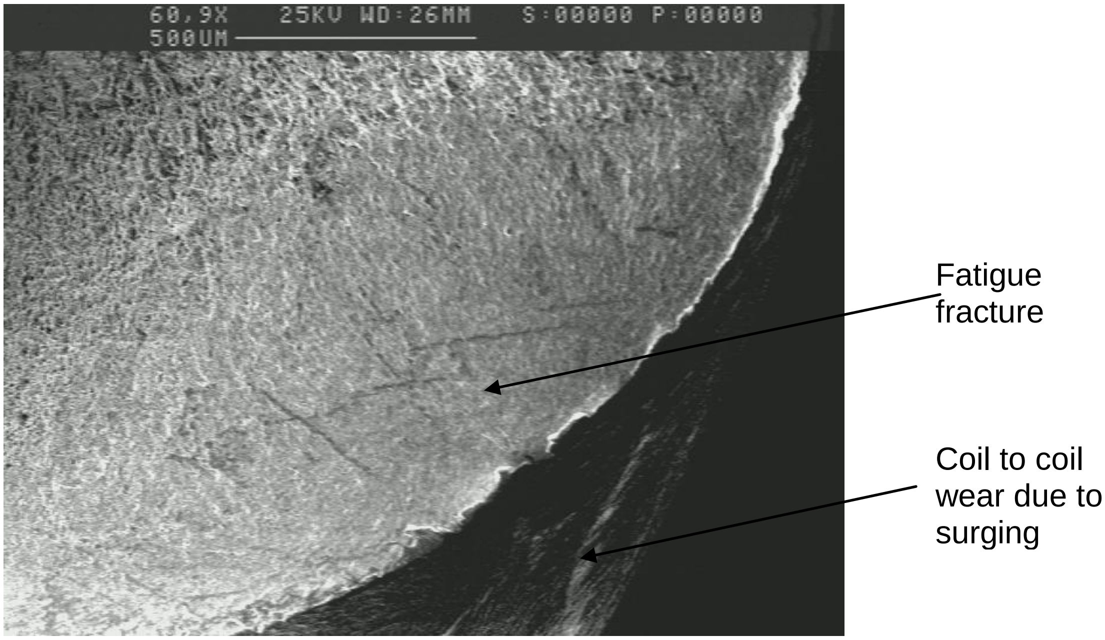

Figure 1 x 60.9

IST has come across a number of instances in which this distinction was not recognised by the designer, the wrong formula was used and natural frequencies were predicted which appeared to be in a harmlessly high speed range, which would not be achieved in service. In reality, unfortunately, the combined spring/mass natural frequency, correctly predicted by the formula given above, did occur within the service speed range and premature failure resulted due to fatigue cause by resonance.

The moral of this cautionary tale is that an understanding of the resonance of your compression springs and your springs as part of a spring/mass system will help you to ensure that they perform as well in several years time as they did on the day you made them.

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