TECHNICALLY SPEAKING

HOOKE'S LAW

Robert Hooke, an English scientist, was the first to recognise that a graph plotted of the weight applied to a wire against its deflection was a straight line. This simple law, dating back to 1680, governs the load output of springs and is fundamental to the whole of the spring industry.

For compression springs, the load and length are proportional to one another and the slope of the line is defined as the spring rate. If the spring ends aren't quite parallel, then the rate will be low until the spring has been deflected sufficiently to make the ends parallel. Similarly, if there is a gap beneath the end tip at one or both ends, then the rate will be low until the gap closes. At the other extreme, as the compression spring is deflected towards its closed or block length there becomes an increasing risk that coils will contact each other thereby becoming inactive and reducing the number of active coils. Then the rate will become higher, and spring design standards such as En 13906-1 define a length L_n beyond which a compression spring should not be used just to avoid this risk.

Hence the theoretical load/length, rate/length graph for a compression spring is as shown schematically in Figure 1.

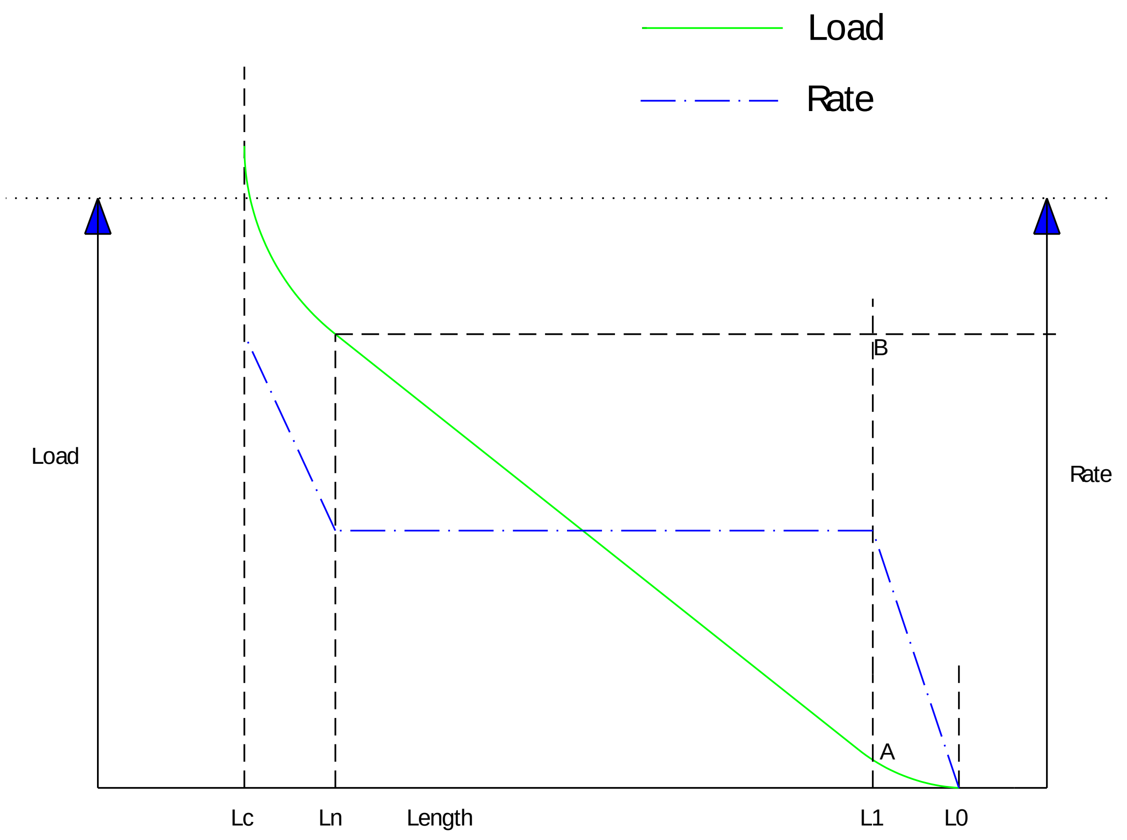


Figure 1 Theoretical Load / Length graph for a compression spring

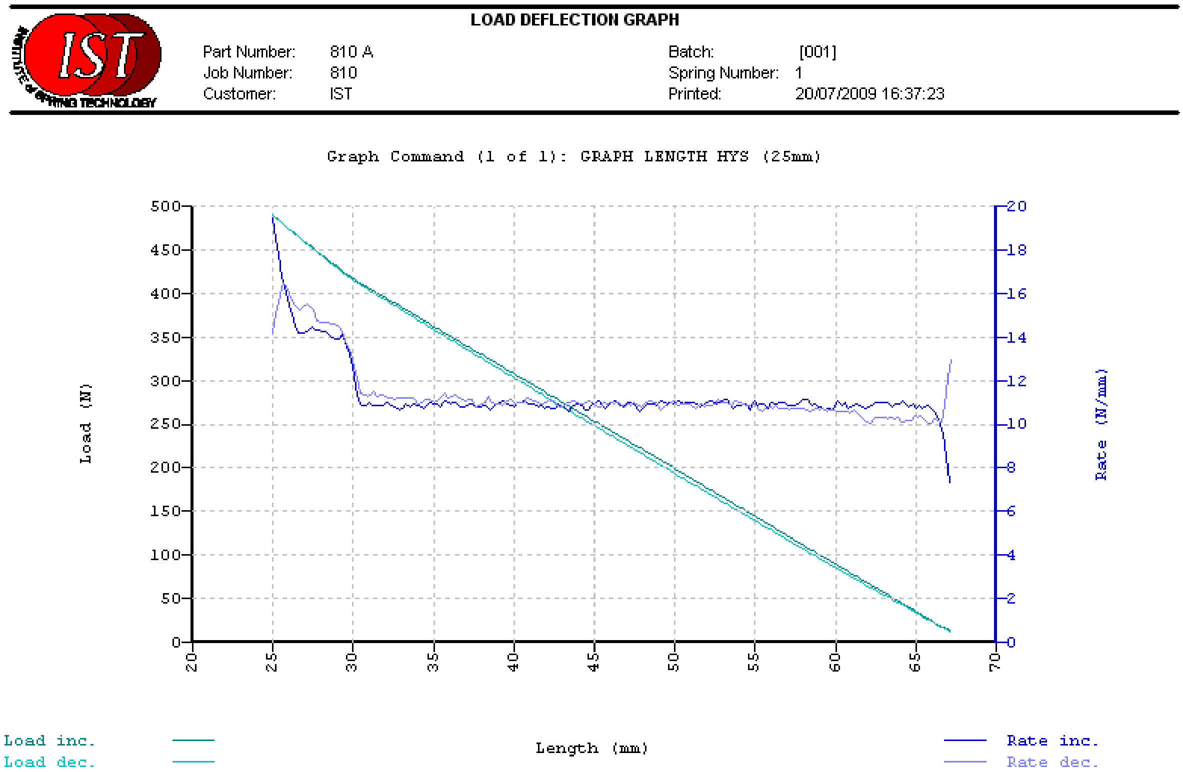
Three correspondents have recently suggested to IST that Hooke's Law is too simple, and that more sophisticated rules are required. The first said that the outside diameter changes as the spring is loaded and this is why the rate is sometimes non-linear between A and B.

The second argued that spring rate was lower after springs had relaxed in service at modestly elevated temperature.

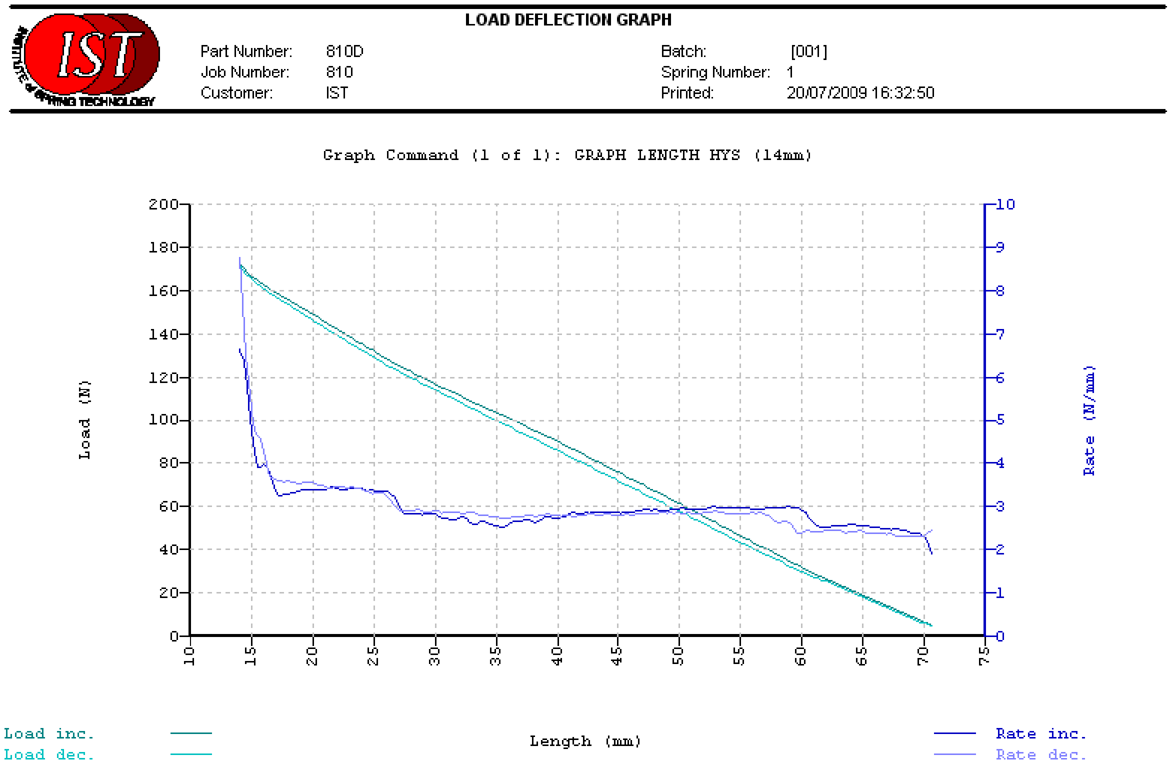
The third argued that the torsional modulus changed during prestressing, and this caused a non-linear rate.

To test these arguments a selection of compression springs (all of which had been proved by IST to have satisfactory dynamic performance) were tested accurately. The objective was to identify whether there was reason to doubt that Hooke's Law was as valid today as it was in 1680. Some of the springs had a non-linear rate as the results in the table below shows:

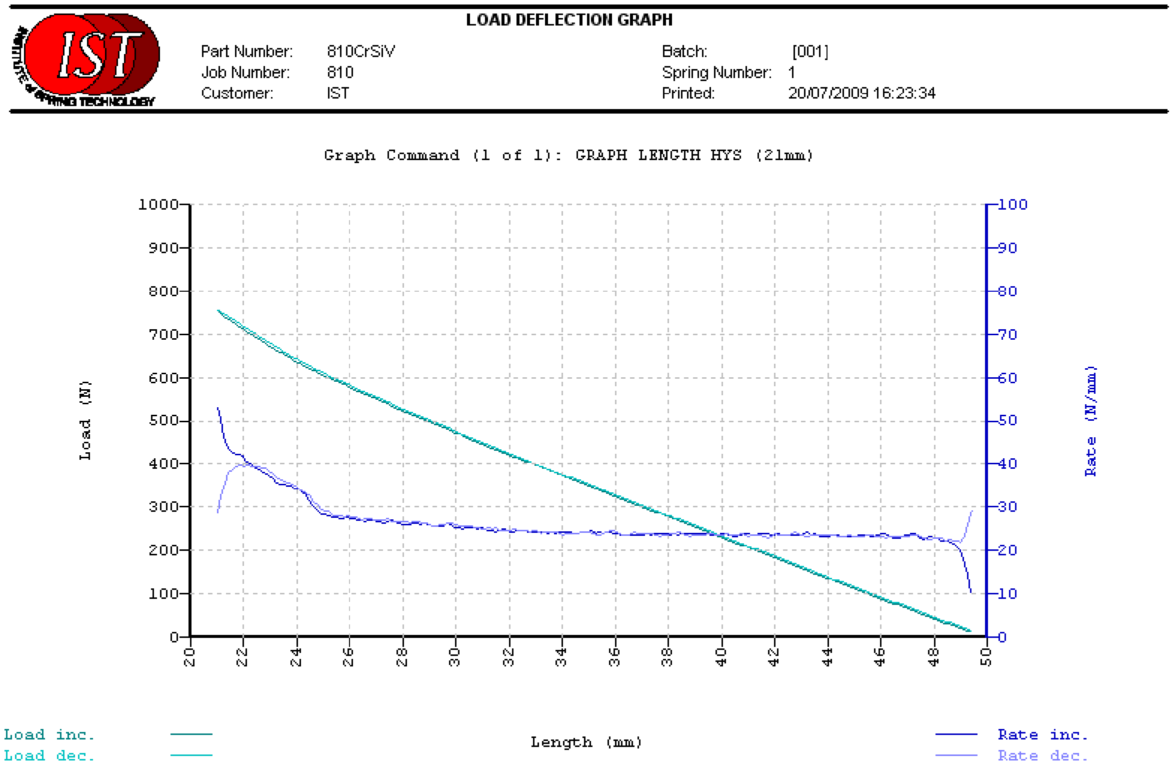
Material	**d/mm mm**	**De mm**	**nt**	**τc (corrected) % UTS**	**R N/mm**	**Rate linear between A & B / % deflection available**
Hastelloy C276	1.61	12.10	7	60	10.0	8-70%
17/7PH	4.88	58.4	10.9	27	4.1	4-51%
302(2)	4.46	47.8	5.75	47	10.7	5-85% Graph 1
SiCr	3.75	22.8	5.6	57	77.9	12-70%
Carbon Steel 1	2.64	24.15	8.9	61	6.8	2-70%
Carbon Steel 2	2.34	31.68	5.5	63	2.85	19-78% Graph 2
302(1)	3.18	43.05	5	57	4.13	2- %
302(1) prestressed	3.18	43.25	5	54	4.08	2-43%
302(1) relaxed	3.18	43.15	5	55	4.10	2-43%
CrSiV	3.46	27.87	6.1	61	24.0	2-80% Graph 3



Graph 1 Example of spring with linear rate from 2 to 85% of the available deflection



Graph 2 Example of spring with rising rate until gap beneath end tip closes (at about 60mm), then a reducing rate as the outside diameter increases (evident in this example because spring has large index and large helix angle). Finally another rate step occurs (at about 27mm) when an active coil closes well before reaching the closed length, and this may also be before Ln.



Graph 3 Example of spring with a gradually rising rate due to number of active coils reducing progressively more quickly than the outside diameter is increasing.

Conclusion

No evidence for rate being non-linear, other than for geometric reasons, was identified. Large index, high helix angle springs will be subject to significant diameter increases, and hence rate reductions. Gentle end coil lay-on will produce a reduction in the number of active coils as the spring is loaded, and hence an increasing rate. Relaxation and prestressing will not invalidate the assumption that ‘Hooke’s Law rules OK’. However, it is reasonable to observe that springs that pass fatigue testing at IST are not as accurately made as theory would predict.

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Readers are encouraged to contact him with comments about this article, and with subjects that they would like to be addressed in future. Contact Hayes at m.hayes@springexpert.co.uk.